

Tumbling toast, Murphy's Law and the fundamental constants

Robert A J Matthews

Department of Applied Mathematics and Computer Science, University of Aston, Birmingham B4 7ET UK†

Received 20 February 1995, in final form 31 March 1995

Abstract. We investigate the dynamics of toast tumbling from a table to the floor. Popular opinion is that the final state is usually butter-side down, and constitutes *prima facie* evidence of Murphy's Law ('If it can go wrong, it will'). The orthodox view, in contrast, is that the phenomenon is essentially random, with a 50/50 split of possible outcomes. We show that toast does indeed have an inherent tendency to land butter-side down for a wide range of conditions. Furthermore, we show that this outcome is ultimately ascribable to the values of the fundamental constants. As such, this manifestation of Murphy's Law appears to be an ineluctable feature of our universe.

Résumé. Nous examinons la dynamique du toast dans sa chute de la table au plancher. L'avis populaire tient à ce que le toast tombe habituellement côté beurré par terre et que cela constitue le commencement de preuve de la loi de Murphy (loi de la guigne maximum). En revanche, l'avis orthodoxe insiste qu'il s'agit d'un phénomène essentiellement dû au hasard, dont les résultats possibles se divisent 50/50. Nous montrons que le toast a, en effet, une tendance fondamentale à arriver côté beurré par terre dans des circonstances diverses et variées. De plus, nous montrons que ce résultat s'attribue en dernière analyse aux valeurs des constantes fondamentales. En tant que tel, cet exemple de la loi de Murphy semblerait être une caractéristique inéluctable de notre univers.

1. Introduction

The term Murphy's Law has its origins in dynamical experiments conducted by the US Air Force in the late 1940s involving an eponymous USAF captain [1]. At its heart lies the concept that 'if something can go wrong, it will'; this has its analogues in many other cultures [2], and is almost certainly of much older provenance.

The phenomenon of toast falling from a table to land butter-side down on the floor is popularly held to be empirical proof of the existence of Murphy's Law. Furthermore, there is a widespread belief that it is the result of a genuine physical effect, often ascribed to a dynamical asymmetry induced by one side of the toast being buttered.

Quite apart from whether or not the basic observation is true, this explanation cannot be correct. The mass of butter added to toast (~ 4 g) is small compared to the mass of the typical slice of toast (~ 35 g), is spread thinly, and passes into the body of the toast. Its contribution to the total moment of inertia of the toast—and thus its effect on the toast's rotational dynamics—is thus negligible.

Similarly, the aerodynamic effect of the thin layer of butter cannot contribute a significant dynamical asymmetry. It is easily shown that for air resistance to contribute significantly to the dynamics of the falling toast, the height of fall must be of the order of $2(\rho_T/\rho_A)d$, where ρ_T is the density of the toast, d is its thickness and ρ_A the density of air. The presence of butter will contribute only a small fraction of this total; supposing it to be a generous 25 per cent and taking the typical values of $\rho_T \sim 350 \text{ kg m}^{-3}$, $\rho_A = 1.3 \text{ kg m}^{-3}$ and $d \sim 10^{-2}$ m, we find that the toast would have to fall from a height over an order of magnitude higher than the typical table for the butter to have significant aerodynamic effects.

Such estimates lend credibility to the widespread 'orthodox' answer to the tumbling toast question: that it is essentially a coin-tossing process in which only the bad outcomes are remembered. Indeed, there is some experimental evidence to support this. In tests conducted for a BBC-TV programme on Murphy's Law [1], buttered bread was tossed into the air 300 times in a variety of situations designed to reveal the presence of Murphy's Law. In all tests, the results were statistically indistinguishable from the 50/50 outcome expected from random coin-tossing, suggesting that selective memory is the true explanation of Murphy's Law.

† Address for correspondence: 50 Norreys Road, Cumnor, Oxford, OX2 9PT UK; email 100265.3005@compuserve.com

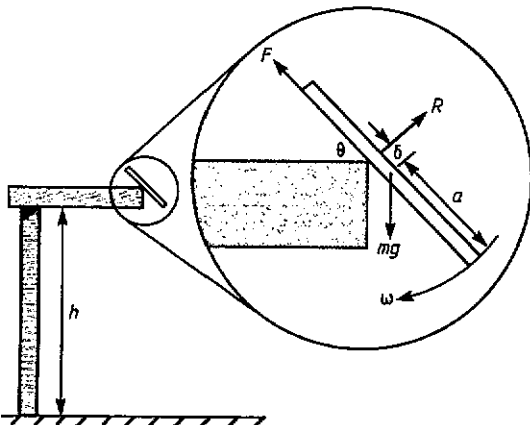
There are, however, two problems with this. First, by its very nature Murphy's Law might contrive to ruin any overt attempt to demonstrate its existence by such probabilistic means. This would make experimental verification of its existence very problematic. A simple Bayesian probability analysis shows that there are grave difficulties with attempts to demonstrate Murphy's Law if it is considered to be a skewing of an otherwise symmetric probability distribution in the direction of an unfavourable outcome. Second, and more seriously, Murphy's Law may be far more fundamental than a skewing of probability distributions: it may actually forbid certain favourable outcomes from taking place. In the case of falling toast, this implies that Murphy's Law might influence the dynamics of the toast at a fundamental yet subtle level. If so, failure to reveal its presence by carelessly hurling toast randomly into the air would hardly be surprising.

As we now show, the dynamics of falling toast are indeed rather subtle, and do depend fairly critically on initial conditions. Nevertheless, in a broad range of realistic circumstances, the dynamics do lead to a bias towards a butter-side down final state. We provide both theoretical and experimental evidence for this conclusion and show that the results have surprisingly deep origins. Specifically, we show that the fall of toast is a manifestation of fundamental aspects of the nature of our universe.

2. Dynamics of falling toast

In what follows we model the tumbling toast problem as an example of a rigid, rough, homogeneous rectangular lamina, mass m , side $2a$, falling from a rigid platform set a height h above the ground. We consider the dynamics of the toast from an initial state where its centre of gravity overhangs the table by a distance δ_0 , as shown in figure 1. Initially, we ignore the process by which the toast arrives at this state,

Figure 1. The initial orientation of the rotating toast.



and also assume that it has zero horizontal velocity; the important effect of a non-zero horizontal velocity is addressed later. Finally we assume a perfectly inelastic impact with the floor with zero rebound.

With these assumptions, the dynamics of the lamina are determined by the forces shown in figure 1: the weight, mg , acting vertically downward, the frictional force, F , parallel to the plane of the lamina and directed against the motion, and the reaction of the table, R . The resulting angular velocity about the point of contact, ω , then satisfies the differential equations of motion

$$m\delta\dot{\omega} = R - mg \cdot \cos \theta \tag{1}$$

$$m\delta\omega^2 = F - mg \cdot \sin \theta \tag{2}$$

$$m(k^2 + \delta^2)\dot{\omega} = -mg\delta \cdot \cos \theta \tag{3}$$

where k is the appropriate radius of gyration, such that $k^2 = a^2/3$ for the rectangular lamina considered here. Multiplying (3) by 2ω and integrating from the initial conditions $\omega = 0$ at $\theta = 0$ leads to:

$$\omega^2 = (6g/a) \cdot [\eta/(1 + 3\eta^2)] \cdot \sin \theta \tag{4}$$

where we have used $\delta \equiv \eta a$, with η ($0 < \eta \leq 1$) being the 'overhang parameter'. Equation (4) is the central equation of the tumbling toast problem, as it gives the rate of rotation of the toast once it has detached from the table from a specific state of overhang. Unless the toast can complete sufficient rotation on its descent to the floor to bring the buttered side facing upwards, the toast will land buttered-side down. Thus if the toast begins its descent at an angle ϕ to the horizontal, then for it to land butter-side up again we must have

$$\omega_0\tau > (3\pi/2) - \phi \tag{5}$$

where ω_0 is the free-fall δ rotation rate and τ the free-fall time for the height of the table h , so that

$$\tau = [2(h - 2a)/g]^{1/2} \tag{6}$$

The frictional force acting on the lamina will prevent detachment until the lamina has rotated through at least an angle ϕ , at which point slipping occurs. This minimum value of ϕ follows from the usual condition $F = \mu R$, where μ is the coefficient of static friction between the lamina and the table edge. From (1), (2) and (4) we find

$$\phi > \arctan [\mu/(1 + 9\eta^2)] \tag{7}$$

To calculate the free-falling angular rotation rate ω_0 , we must deal with the post-slipping regime. At the instant of slipping, the centre of rotation of the lamina is a distance $a\eta$ from the centre of gravity, and the rotational rate is given by (4). A point on the shorter, non-overhanging section of lamina at a distance $a(\eta + \epsilon)$, $0 < \epsilon \ll 1$ from the CG will thus have a rotationally-induced horizontal component of velocity $a\epsilon\omega \cdot \sin \phi$ away from the table. Slipping will bring this point vertically over the table edge, so

that contact between table and toast is broken, the latter then tumbling about its CG at a rotational rate ω_0 essentially unchanged from the original value. Although irregularity in the surface of the toast can prevent immediate post-slip detachment, experiments confirm that the value of ω_0 can be taken as that induced by the initial overhang torque of $mg\alpha\eta_0$. Thus the free-falling toast rotates at a rate

$$\omega_0^2 = (6g/a) \cdot [\eta_0 / (1 + 3\eta_0^2)] \sin \phi \quad (8)$$

where the value of the critical overhang parameter η_0 and slip angle ϕ at which detachment takes place may be determined experimentally. To place a lower limit on the overhang needed to avoid a butter-side down final state, we insert (8) in (5), set $\phi = \pi/2$ and solve the resulting quadratic equation for η_0 :

$$\eta_0 > \frac{1 - \sqrt{[1 - 12\alpha^2]}}{6\alpha} \quad (9)$$

where $\alpha \equiv \pi^2/12(R - 2)$ and $R \equiv h/a$

For conventional tables and slices of toasts, we have $h \sim 75$ cm, $2a \sim 10$ cm leading to $R \sim 15$, $\alpha \sim 0.06$ and thus a lower limit on the critical overhang parameter of

$$\eta_0 > 0.06 \quad (10)$$

if the toast is to complete sufficient rotation to avoid a butter-side down final state.

3. Experimental results and implications

An experimental determination of η_0 holds the key to establishing whether or not the fall of toast constitutes a manifestation of Murphy's Law. Tests were carried out using a lamina derived from a standard white loaf (supplied by Michael Cain & Co., Oxford Road, Cumnor, Oxford). The lamina was cut into a rectangle of 10 cm \times 7.3 cm \times 1.5 cm (so that $2a = 10$ cm), and placed on a rigid flat and level platform of kitchen Contiboard, used to model the surface of a clean, uncovered table.

Measurements of the value of the coefficient of static friction μ between the lamina and the platform were made by measuring the angle of the platform at which sliding just began; the tangent of this angle is then equal to μ . Test were carried out on both bread and toast, leading to

$$\begin{aligned} \text{For bread: } [\mu]_{\text{obs}} &\sim 0.29 \\ \text{For toast: } [\mu]_{\text{obs}} &\sim 0.25 \end{aligned} \quad (11)$$

Measurements of the value of the critical overhang parameter η_0 were then made by placing the lamina over the edge of the Contiboard and determining the least amount of overhang of the $2a = 10$ cm edge at which detachment and free-fall took place.

This was found to be

$$\begin{aligned} \text{For bread: } [\eta_0]_{\text{obs}} &\sim 0.02 \\ \text{For toast: } [\eta_0]_{\text{obs}} &\sim 0.015 \end{aligned} \quad (12)$$

Both bread and toast are thus relatively unstable to tumbling from overhanging positions. Crucially, neither can sustain overhangs anywhere near as large as the critical value given in (10). This implies that laminae with either composition do not have sufficient angular rotation to land butter-side up following free-fall from a table-top. In other words, the material properties of slices of toast and bread and their size relative to the height of the typical table are such that, in the absence of any rebound phenomena, they lead to a distinct bias towards a butter-side down landing. But before this can be taken as confirmation of popular belief, however, some practical issues must be addressed.

4. The effects of non-zero horizontal velocity

So far, we have ignored the means by which the toast comes to be in the overhang condition shown in figure 1. This is clearly of practical importance, however, as the toast will typically leave the table as the result of sliding off a tilted plate, or being struck by a hand or arm. The consequent horizontal velocity may dominate the dynamics if the gravitational torque has insufficient time to induce significant rotation. In this case, the toast will behave like a simple projectile off the edge of the table, keeping its butter-side up throughout the flight. This raises the possibility that, while dynamically valid, the butter-side down phenomenon may only be witnessed for an infeasibly small range of horizontal velocities. To investigate this range, we first note that the time for an initially horizontal lamina of overhang parameter η to acquire inclination ψ follows from (8):

$$t(\psi) = [a(1 + 3\eta_0^2)/6g\eta_0]^{1/2} \cdot I(\psi) \quad (13)$$

where

$$\begin{aligned} I(\psi) &= \int_0^\psi d\theta / (\sin \theta)^{1/2} \sim \int_0^\psi d\theta / \theta^{1/2} \\ &= 2\psi^{1/2} \quad \text{for small } \psi \end{aligned} \quad (14)$$

If the lamina has a horizontal velocity V_H as it goes over the edge of the table, the time during which it is susceptible to torque-induced rotation is $\sim a/V_H$. During this time its average overhang parameter η_0 will be of the order 0.5, and it will acquire a downward tilt through the torque of order ψ . If this angle is small, the dynamics of the lamina can be considered those of a projectile. By (13) and the small angle approximation in (14), this implies that the effects of torque-induced rotation, and thus tumbling motion, will be negligible for horizontal

velocities above about

$$V_H \sim (3ga/7\psi)^{1/2} \sim 1.6 \text{ m s}^{-1} \text{ (with } \psi \sim 5^\circ) \quad (15)$$

At speeds considerably below this value (below, say, $V_H/5 \sim 350 \text{ mm s}^{-1}$) the torque-induced rotation should still dominate the dynamics of the falling toast, and the butter-side down phenomenon should still be observed. This conclusion is supported by observation. Furthermore, the relatively high value of V_H ensures that the butter-side down phenomenon will be observed for a wide range of realistic launch scenarios, such as a swipe of the hand or sliding off an inclined plate (which, by (11), will have to be tilted downward by at least $\sim \arctan(0.25) \sim 14^\circ$).

It therefore appears that the popular view that toast falling off a table has an inherent tendency to land butter-side down is based in dynamical fact. As we now show, however, this basic result has surprisingly deep roots.

5. Tumbling toast and the fundamental interactions

We have seen that the outcome of the fall of toast from a table is dictated by two parameters: the surface properties of the toast, which determine η_o , and the relative dimensions of the toast and table, which determine R . The latter is, of course, ultimately dictated by the size of humans. Using an anthropic argument, Press [3] has revealed an intriguing connection between the typical height of humans and the fundamental constants of nature. It centres on the fact that bipedal organisms like humans are intrinsically less stable than quadrupeds (e.g. giraffes), and are more at risk of death by toppling. This leads to a height limitation on humans set by the requirement that the kinetic energy injected into the head by a fall will be insufficient to cause major structural failure and death. This height limitation on humans in turn implies a limit on the height of tables. We now deduce this limit using an anthropic argument similar to that of Press.

We begin by considering a humanoid organism to be a cylindrical mass of polymeric material of height L_H whose critical component is a spherical mass M_C (the head) positioned at the top of the body. Then, by Press's criterion, the maximum size of such an object is such that

$$f \cdot (M_C v_{\text{fall}}^2/2) < NE_B \quad (16)$$

where $v_{\text{fall}} \sim \sqrt{3gL_H}$ is the fall velocity, $f(\sim 0.1)$ is the fraction of kinetic energy that goes into breaking N polymeric bonds of binding energy E_B , and the fracture is assumed to take place across a polymer

plane $n(\sim 100)$ atoms thick, so that

$$N \sim n(M_C/m_p)^{2/3} \quad (17)$$

where m_p the mass of the proton. Thus the height of the humanoid will be of the order

$$L_H \sim (n/f)(M_C/m_p)^{2/3} \cdot E_B/M_C g \quad (18)$$

A simple Bohr-atom model shows that

$$E_B \sim q\alpha^2 m_e c^2 \quad (19)$$

where α is the electronic fine structure constant, m_e the mass of the electron, c the speed of light, and q for polymeric materials is $\sim 3 \times 10^{-3}$. The acceleration due to gravity, g , for a planet can also be estimated from first principles, using an argument based on balancing internal gravitational forces with those due to electrostatic and electron degeneracy effects [4]. This leads to

$$g \sim (4\pi G/3\mu^3)(\alpha/\alpha_G)^{1/2} m_p/a_0^2 \quad (20)$$

where $\mu(\sim 6)$ is the radius of the polymeric atoms in units of the Bohr radius a_0 , and α_G is the gravitational fine structure constant $Gm_p^2/\hbar c$. We also have

$$M_C \sim 4\pi R_C^3 \rho_o/3 \quad (21)$$

where R_C is the radius of the critical component ($\sim L_H/20$) and ρ_o is the atomic mass density

$$\rho_o \sim Am_p/(\mu a_o)^3 \quad (22)$$

where $A(\sim 100)$ is the atomic mass of the polymeric material. Substituting these relations into our original criterion for L_H gives, after some reduction,

$$L_H < K \cdot (\alpha/\alpha_G)^{1/4} \cdot a_o \quad (23)$$

$$\text{where } K \equiv (3ng/f)^{1/2} \mu^2 A^{-1/6} \sim 50$$

Inserting the various values, we find that this first-principles argument leads to a maximum safe height for human of around 3 metres. Although the estimate of L_H is pretty rough and ready, its weak dependency on the uncertainties in the various factors in (23) makes it fairly robust. The resulting limit has a number of interesting features. The estimate of its value agrees well with the observation that a fall onto the skull from a height of 3 m is very likely to lead to death; interestingly, even the tallest-ever human, Robert Wadlow (1918–1940), was—at 2.72 m—within this bound. The limit on height is also universal, in that it applies to all organisms with human-like articulation on any planet. Most importantly, however, it puts an upper limit on the height of a table used by such organisms: around $L_H/2$, or 1.5 m. This is about twice the height of tables used by humans, but still only half that needed to avoid a butter-side down final state for

tumbling toast: rearranging (9) we find

$$R = 2 + \frac{\pi^2(1 + 3\eta^2)}{12\eta} \quad (24)$$

and inserting the observed value $\eta \sim 0.015$ given in (12) leads to $R \sim 60$ and $h \sim 3$ metres. The limit (23) thus implies that all human-like organisms are doomed to experience tumbling toast landing butter-side down.

6. Conclusions

Our principal conclusion is a surprising one, given the apparently quotidian nature of the original phenomenon: all human-like organisms are destined to experience the 'tumbling toast' manifestation of Murphy's Law because of the values of the fundamental constants in our universe. As such, we have probably confirmed the suspicions of many regarding the innate cussedness of the universe. We therefore feel we must conclude this investigation on a more optimistic note. What can human-like—and thus presumably intelligent—organisms do to avoid toast landing butter-side down?

Building tables of the ~ 3 m height demanded by (24) is clearly impracticable. Reducing the size of toast is dynamically equivalent, but the required reduction in size (down to squares ~ 2.5 cm across) is also unsatisfactory.

The best approach is somewhat counter-intuitive: toast seen heading off the table should be given a smart swipe forward with the hand. Similarly, a plate off which toast is sliding should be moved swiftly downwards and backwards, disconnecting the toast from the plate. Both actions have the effect of minimising the amount of time the toast is exposed to the gravitationally-induced torque, either by giving the toast a large (relative) horizontal velocity or by sudden disconnection of the point of contact. In both cases, the toast will descend to the floor keeping the butter side uppermost.

We end by noting that, according to Einstein, God is subtle, but He is not malicious. That may be so, but His influence on falling toast clearly leaves much to be desired.

Acknowledgements

It is a pleasure to thank Professor Ian Fells and Robin Bootle for providing background on Murphy's Law.

References

- [1] Bootle R and Fells I 1991 *QED: Murphy's Law* (London, BBC)
- [2] Bootle R 1995 personal communication
- [3] Press W H 1980 *Am. J. Phys.* **48** 597–8
- [4] Davies P C W 1982 *The Accidental Universe* (Cambridge: Cambridge University Press) 44–9