Murphy’s Law of Maps

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Summary
Using probability theory, I show that the “urban myth” that map locations tend to lie on the edges or down the central crease is based in fact.

◆MURPHY’S LAW◆

ONE of the banes of everyday life is having to deal with the consequences of Murphy’s Law: “If something can go wrong, it will”. First coined almost 50 years ago by an eponymous US Air Force Captain working on rocket sled experiments, Murphy’s Law is popularly blamed for everything from toast landing butter-side down to the impossibility of finding matching pairs of socks in the morning.

Despite the wealth of anecdotal evidence for the validity of Murphy’s Law, however, most scientists still regard it as no more than a silly urban myth. Indeed, a programme in the BBC-TV popular science series *Q.E.D.* in 1993 is often cited as having debunked the concept of Murphy’s Law. For example, in tests involving tossing bread into the air hundreds of times, it emerged that the buttered side landed face-down just as often as face-up. These tests were, however, fundamentally misconceived: people do not usually toss bread or toast into the air before eating it. A more plausible route for toast to end up on the floor is via sliding off a plate or being nudged off a table – processes dynamically completely different from a coin-toss. It turns out that under these more plausible conditions, the resulting rigid-body dynamics *do* lead toast to have a bias towards butter-down landings (Matthews 1995). I have since found that many other notorious manifestations of Murphy’s Law are equally well-founded, with explanations ranging from combinatorics for the plethora of odd socks to knot theory in the case of tangled rope (Matthews 1995, 1996a,b,c, 1997).

In what follows, I investigate a phenomenon very familiar to those who make heavy use of maps and road atlases: the apparent predilection of places we are looking for to lie in awkward parts of the map, such as along the edge or down the central crease. I show that this Murphy’s Law of Maps – “If a location can lie in an awkward part of the map, it will” – is no mere urban myth, but can be explained using geometry and probability theory.

Let us take our map to be rectangular, of sides length m and n, ($m > n$; see Figure 1). We can then define a “Murphy Zone” around the edge of the map, and to either side of the central crease, of width b ($<n/2$, to prevent the Murphy Zone overlapping itself).

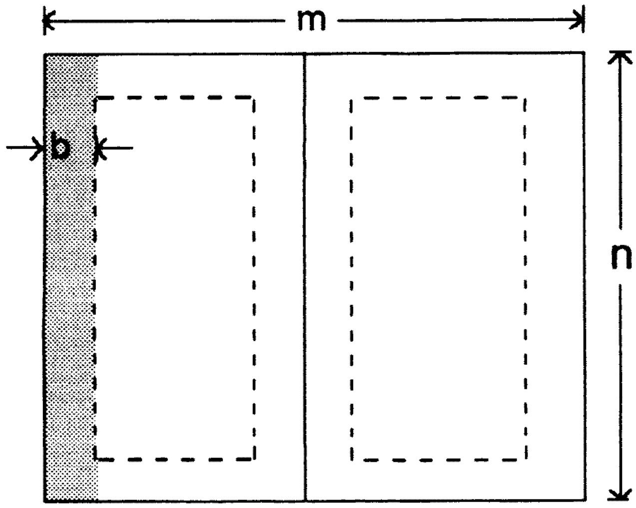

Figure 1 The "Murphy Zones" on a typical map

Simple geometry then shows that the total area of the map that falls into this Murphy Zone is A, where

$$A=2b(2n+m-4b). \qquad (1)$$

As the total area of the map is simply $m \times n$, the probability that a point picked at random will be in the Murphy Zone is P, where

$$P=A/mn=2b(2/m+1/n-4b/mn). \qquad (2)$$

For simplicity, let us now take the case of a square map, for which $m = n$. Then (2) becomes

$$P = 6(b/m) - 8(b/m)^2 \qquad (3)$$

The Murphy Zone shown on Figure 1 takes up just one-tenth of the total page width; henceforth, we will take this as the definition of the Murphy Zone. Now comes the surprise. What is the probability of a destination picked at random lying within this area? Setting $b/m = 0.1$ in (3) we find that

$$P = 0.52. \qquad (4)$$

In other words, a point picked at random has better than 50:50 odds of ending up in a Murphy Zone of width just one-tenth that of the whole map. At first sight, this seems very surprising; mathematically, the explanation is simply that the Zone tracks the outermost – and thus largest – dimensions of the map, so only a relatively narrow width still encloses a comparatively large total area.

Those who still doubt the result should carry out the ultimate test of any explanation, and perform an experiment: use a random number generator to pick, say, 100 random locations from the index at the back of most atlases, and count the proportion that fall into the Murphy Zone leading to (4).

Having established the root cause of Murphy's Law of Maps, we can ask a number of related questions, such as the probability of both our starting point and our destination lying in a Murphy Zone. Again, this is surprisingly high: for the size of Murphy Zone given above, and assuming both locations are randomly distributed, the probability is 0.52 x 0.52 = 0.27 - in other words, over one in four of all our trips will both begin and end in a Murphy Zone.

These results are, of course, based on two assumptions: firstly, that map locations are randomly distributed within the map area, and secondly that the map is square. It is hard to see why an *individual* location on a map should not be randomly distributed; map page areas are typically defined with reference to the Ordnance Survey Grid, and these bear no obvious relation to human habitation patterns. While some maps may try to centre on a major conurbation such as Birmingham, the haphazard distribution of others will still tend to randomise their locations. That said, if the map is so large-scale that our journeys from place to place span very little distance, then our assumption that both starting point and destination are independent random variables is decidedly shaky. Thus one should not put too much faith in the 27 per cent figure given above.

The assumption that the map is square is much more important, as most road atlas pages are not square, but typically have an "aspect ratio" $K = m / n$ of around 1.4. As we shall now show, K holds the key to combating Murphy's Law of Maps.

◆COMBATING MURPHY'S LAW◆ OF MAPS

In considering maps with aspect ratios $K > 1$, we must be slightly more careful in our definition of the width of the Murphy Zone relative to m and n, the leading dimensions of the map. There are two advantages in defining it relative to n, the shorter of the two dimensions. Firstly, it is then easier to ensure we do not breach the condition $b < n/2$ needed to avoid any Murphy Zones overlapping each other. Second, it prevents a Murphy Zone that is relatively thin compared to the edges of length m producing a ludicrously thick one relative to the edges of length n. So, defining $r = b/n$, equation (2) becomes

$$P = [(4/K) + 2]r - (8/K)r^2 \qquad (5)$$

Setting $r = 1/10$ and $K = m/n = 1.4$, we find $P = 0.43$, compared to P = 0.52 for a square map. Thus, increasing the aspect ratio K *reduces* the chances of our landing in the Murphy Zone (essentially because the proportion of it running parallel to the longest sides, m, tends to zero). This feature of map design does not appear to have been widely recognised by map-makers. The *Reader's Digest Atlas of the British Isles* (Reader's Digest Association Ltd., 1988) has a relatively large aspect ratio, because of its "exclusive" fold-out flaps which increase the aspect ratio from around K = 1.37 ($P = 0.43$) to $K = 1.54$, reducing the chances of landing in the Murphy Zone to 0.41. However, these flaps were apparently introduced merely to allow roads to be followed easily from one page to another and they only work on one side of the map; if they worked on both, they would reduce P to just 0.38.

While changing the aspect ratio of maps would help ameliorate the worst effects of Murphy's Law of Maps, it is impossible to evade its effects completely: from (5) we see that even as K tends to infinity, P approaches the asymptotic value of 2r - which is small (0.20 for $r = 1/10$), but still non-zero!

◆CONCLUSION◆

Scientists are often quick to dismiss popular beliefs like Murphy's Law as nothing more than "urban myths". However it is often worth pausing to wonder precisely why so many people believe in a particular phenomenon. Are they really all dunderheads who just forget all the times the phenomenon does *not* occur? Or might there be some deeper explanation, based on counter-intuitive probabilistic arguments? In the case of Murphy's Law of Maps, there is a particularly simple explanation for why people think map locations tend to lie in awkward places. They do.

More Murphy-related information can be found at: http://ourworld.compuserve.com/homepages/rajm/

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